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A short note on a technique for solving mixing problems by using the Laplace transform

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Abstract

In this paper, technique in a Matlab for solving the mixing problem using the Laplace transformation is provided. In general, a Matlab function has been developed to estimate and compute the solution to a mixing issue. The program's output displays the elapsed time, the solution, and its numbers.

keywords

Mixing problems , Laplace transform , A technique .

1 Introduction

The paper focuses on using the Laplace transform to handle mixed problems related to an ODE. The main goal of this paper is to use Laplace transforms to solve differential equations (ODE) with conditions (mixed problems).

For this, we will attempt to describe a theoretical example in order to gain a practical understanding of how to apply the theory.

To tackle mixed issues connected with ODE, using the Laplace transform. The amount of salt in a mixing tank is a common mixing problem. At a specific rate, salt and water enter the tank, mix with what's already there, and then exit at a particular rate. To model the problem, we'll build a differential equation and solve it.
The time, t, in some appropriate unit, shall be the independent variable (seconds, minutes, etc). It's not always clear whether to use the salt concentration in the liquid or the amount of salt as a dependent variable. It's normally easier to use the quantity, Q. (again, in appropriate units such as kilograms)[[1],[5]].

1.1 definition

The Laplace integral of a function f(t) defined for $0 \leq t < \infty$ is define by

$$\mathscr{L}[f(t)] = \int_0^\infty e^{-st} f(t)dt \tag{1}$$

whenever the improper integral converges.

$$\mathscr{L}[f(t)] = \int_0^\infty e^{-st} f(t)dt = \lim_{k\to\infty} \int_0^k e^{-st} f(t)dt$$

The proper Laplace Transform for a function f (t) is indicated as $\mathscr{L}[f(t)]$, where $\mathscr{L}$ is the operator applied to the time domain function f(t).[4]

1.2 Inverse of Laplace Transform

$\mathscr{L}[f(t)] = F(s)$, then $\mathscr{L}^{-1}[F(s)] = f(t)$ is called inverse Laplace Transform of $F(s)$

2 Mixing problems

The one-compartment system proposes that the derivative of x with respect to time can be read at the rate of change in the amount of a substance in a compartment over time. A function x(t) is used to represent the rate at which a substance enters and exits a compartment.

$$\frac{dx}{dt} = rate \quad in - rate \quad out$$

as a mathematical model of the process. The one-compartment system is an useful representation of the mixing of fluids in a tank. The rates at which this substance enters and exits the tank will be done by calculation . The input rate is calculated by multiplying the flow rate (volume/time) by the concentration (amount/volume).

The intended output rate of the chemical is obtained by multiplying this concentration by the mixture's exit rate. The substance's output rate is frequently more difficult to estimate. By dividing the amount x(t) by the volume of the mixture in the tank at time t, we may calculate the concentration of that chemical.

3 Applications

Example:

In the tank holds 1000 gallons of water in which 100 pound of salt is originally dissolved. Brine is delivered at a rate of 10 gallon per minute, with each gallon containing 5 pound of dissoved salt. Stirring keeps the mixture in the tank consistent. At a rate of 10 gal per minute, the brine will run out. Calculate the amount of salt in the tank at any point in time t.[3]

$$\tfrac{dx}{dt} = \text{Salt inflow rate} - \text{Salt outflow rate}$$

$$\frac{dx}{dt}\left(\frac{pound}{minute}\right) = \left(5\frac{pound}{gal}\right) \times \left(10\frac{gal}{minute}\right) - \left(\frac{x(t)}{1000}\frac{pound}{gal}\right) \times \left(10\frac{gal}{minute}\right) \tag{2}$$

$$\frac{dx}{dt} = 50 - 0.01x(t)$$

Take both sides' Laplace transforms now.

$$\mathscr{L}\left\{\frac{dx}{dt}\right\} + \mathscr{L}\left\{\frac{10x(t)}{1000}\right\} = \mathscr{L}\{50\} \tag{3}$$

$$sx(s) - x(0) + \frac{10}{1000}x(s) = \frac{50}{s}$$

$$sx(s) - 100 + \frac{10}{1000}x(s) = \frac{50}{s}$$

$$(s + 0.01)x(s) = \frac{50}{s} + 100$$

$$x(s) = \frac{50}{s(s + 0.01)} + \frac{100}{s + 0.01} \tag{4}$$

By Partial fraction

$$x(s) = \frac{5000}{s} - \frac{5000}{s + 0.01} + \frac{100}{s + 0.01}$$

On both sides, apply the Inverse Laplace Transform.

$$\mathscr{L}^{-1}\{x(s)\} = \mathscr{L}^{-1}\left\{\frac{5000}{s}\right\} - \mathscr{L}^{-1}\left\{\frac{500}{s + 0.01}\right\} + \mathscr{L}^{-1}\left\{\frac{100}{s + 0.01}\right\} \tag{5}$$

$$x(t) = [5000 - 4900e^{-0.01t}] \tag{6}$$

Example:
A tank contains 200 liters of fluid in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is then pumped into the tank at a rate of 4 L/min; the well-mixed solution is pumped out at the same rate. Find the number x(t) of grams of salt in the tank at time t.[6]
Solution:
$\frac{dx}{dt}$ = Salt inflow rate − Salt outflow rate

$$\frac{dx}{dt} = 4 - 0.02x(t) \tag{7}$$

Take both sides' Laplace transforms now.

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} + \mathcal{L}\left\{\frac{4x(t)}{200}\right\} = \mathcal{L}\{4\} \tag{8}$$

$$sx(s) - x(0) + \frac{1}{50}x(s) = \frac{4}{s} \tag{9}$$

$$sx(s) - 30 + \frac{1}{50}x(s) = \frac{4}{s}$$

$$(s + 0.02)x(s) = \frac{4}{s} + 30$$

$$x(s) = \frac{4}{s(s+0.02)} + \frac{30}{s+0.01} \tag{10}$$

By Partial fraction

$$x(s) = \frac{200}{s} - \frac{200}{s+0.02} + \frac{30}{s+0.02}$$

On both sides, apply the Inverse Laplace Transform.

$$\mathcal{L}^{-1}\{x(s)\} = \mathcal{L}^{-1}\left\{\frac{200}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{200}{s+0.02}\right\} + \mathcal{L}^{-1}\left\{\frac{30}{s+0.02}\right\} \tag{11}$$

$$x(t) = [200 - 170e^{-0.02t}] \tag{12}$$

4 Technique of Solution

The Matlab program is used , For a and b are constant ODE coefficients, x0 is x(0) . After calling the matlplace.m function, the initial conditions and function f(t) must be entered.[2] In Matlab's editor window, the following Matlab Techniques are defined:

```
function [a,b,x0]=laplace(a,b,x0)
syms s t X
f=input('input function f=');
tic
F =laplace(f,t,s);
Xt = s * X - x0;
Sol = solve(a*Xt + b*X-F,X);
disp('The Laplace transfromation solution is ')
x=ilaplace(Sol,s,t)
toc
ezplot(x)
grid on
title('Figure of The Laplace transfromation solution ')
xlabel('t'),ylabel('x(t)')
end
```

When you save the program to Matlab's current folder and run it, it will discover a solution and generate a graph with the elapsed time.

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To return to the example (1)
>>Laplace(1,0.01,100)
input function f=50
The Laplace transformation solution is
$x(t) = 5000 - 4900 * exp(-t/100)$
Elapsed time is 0.075452 seconds.

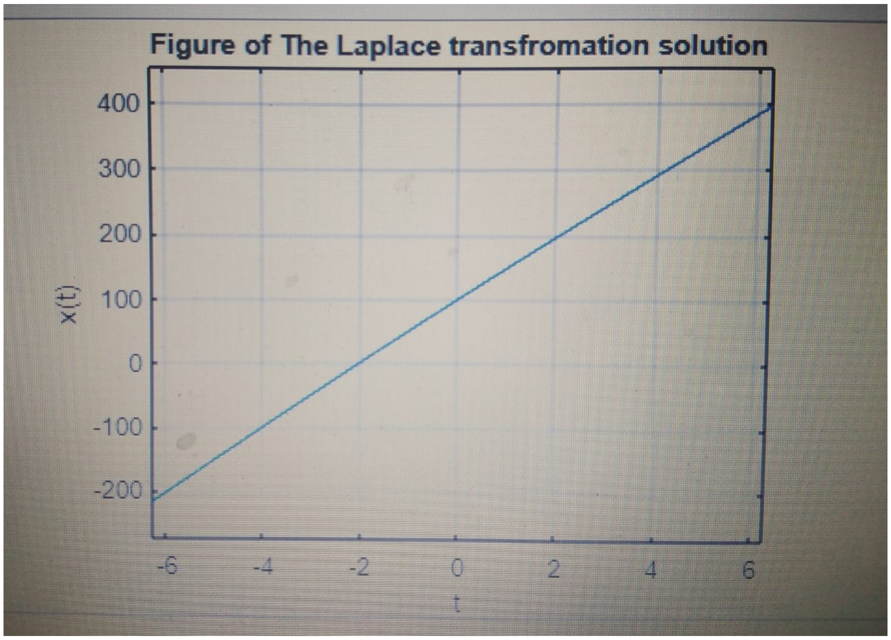

Figure 1: The Laplace transformation solution

To return to the example (1)
>> Laplace(1,0.02,30)
input function f=4
The Laplace transformation solution is
$x(t) = 200 - 170 * exp(-t/50)$
Elapsed time is 0.079348 seconds.

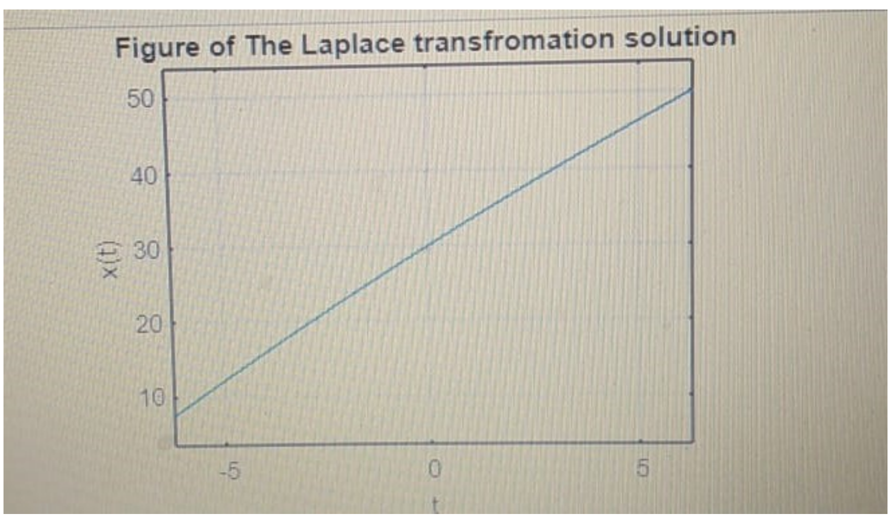

Figure 2: The Laplace transformation solution

Conclusion

We can see the utility of the Laplace Transforms in tackling mixed issues connected with an ODE in this study, which makes the calculations easier. We can only observe the solution to the mixing problem in this case. By differential equations MATLAB was used to solve these equations.
The paper revisit the classic calculus problem of representing brine flow in a system of tanks linked by pipes. We show that the related linear system of differential equations can be solved analytically for various tank layouts including an arbitrary number of tanks. Finally, we look at how solutions for a general closed system of tanks behave asymptotically. The study of the Laplace transform for directed graphs turns out to be strongly related to the problem.

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